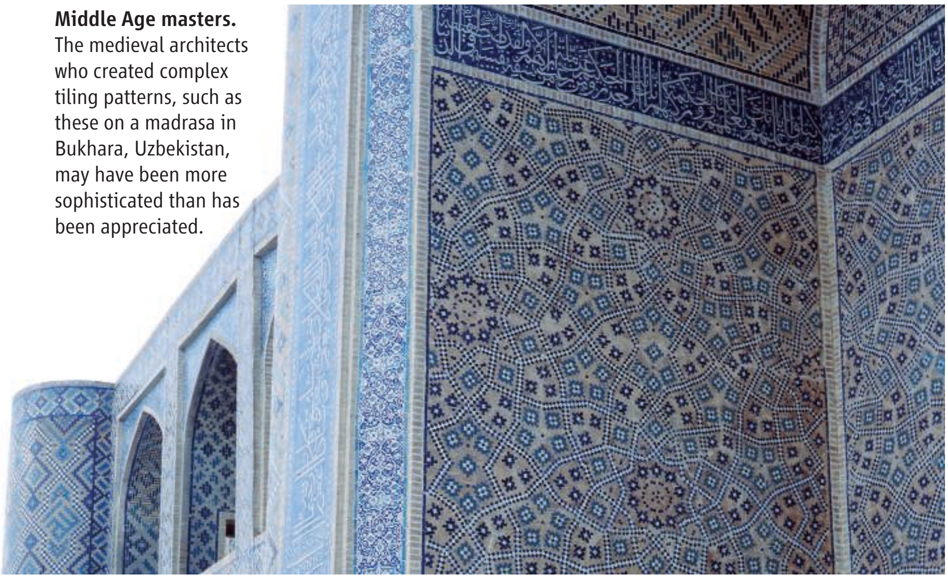
MATHEMATICS

Quasi-Crystal Conundrum Opens a Tiling Can of Worms

The mosques and palaces of the medieval Islamic world are wonders of design. Because tradition forbids any pictorial decorations, they are covered with complex and intricate mosaics. These geometric patterns, called *girih* in Arabic, may be even more sophisticated than has been appreciated.

On page 1106, physicists Peter Lu of Harvard University and Paul Steinhardt of Princeton University propose that architects made a conceptual breakthrough sometime between the 13th and 15th centuries. By first visualizing a surface as a tiling of polygons, these unknown scholars created *girih* patterns that conform almost exactly to a pattern called a quasi-crystal. If Lu and Steinhardt are right, then the Islamic world discovered a piece of mathematics 500 years before it was formally described in the West. But the paper has also sparked a rancorous dispute over who first made this insight, and whether it is true at all.



Middle Age masters. The medieval architects who created complex tiling patterns, such as these on a madrasa in Bukhara, Uzbekistan, may have been more sophisticated than has been appreciated.

Starting in the 1960s, mathematicians studying the geometry of tiling came up with the concept of the quasi-crystal. Tiling is crystalline if it is made up of an infinitely repeating pattern of some finite set of units. Quasi-crystals are also made up of a finite set of interlocking units, but their pattern never repeats even if tiled infinitely in all directions. Researchers also found that although pentagons and decagons don't fit easily into normal tiling, in a quasi-crystal such fivefold and 10-fold rotational symmetries are integral. The most famous quasi-crystal pattern is "Penrose tiling," named after Oxford University mathematician and cosmologist Roger Penrose.

In 2005, Lu, a doctoral student at Harvard, noticed a geometric pattern on the wall of an Islamic school in Uzbekistan with surprisingly complex decagonally symmetric motifs. "It got me thinking that maybe quasi-crystals had been discovered by Islamic architects long ago," he says. Islamic architects began to explore motifs with fivefold and 10-fold rotational symmetry during a flourishing of geometric artistry between the 11th and 16th centuries.

Back at Harvard, Lu began to study architectural scrolls from that period. On many scrolls, faintly sketched beneath the intricate lines of the *girih* design, was a polygonal tiling pattern. "I found the outlines of the same tile shapes appearing over and over," he says. Lu realized that Islamic architects could have used a pattern of polygonal shapes—which he calls *girih* tiles—as the starting point for their designs, creating a wonderfully complex *girih* pattern by tracing lines from tile to tile following local rules. And if the right shape of *girih* tiles were laid together just so, the resulting pattern could be extended forever without repeating—a quasi-crystal.

Lu examined "a few thousand" photos of real mosques and found that although decagonal *girih* patterns became increasingly common from 1200, nearly all are periodic and so are not quasi-crystals. But then he found a photo of the Darb-i Imam shrine in Isfahan, Iran, built in 1453. Its decagonally symmetric motifs on two different length scales are a telltale sign of a quasi-crystal. Working with Steinhardt, his former undergraduate adviser and a quasi-crystal expert, Lu found that the Darb-i Imam *girih* pattern can map onto a Penrose tiling. There were a few defects, but these are superficial, says Lu, and were likely introduced by workers during construction or repair. "We realized that by the 15th century, these architects had the makings of quasi-crystals," says Lu.

The paper has had a mixed reception. Crystal expert Emil Makovicky of the University of Copenhagen, Denmark, studied *girih* patterns for 2 decades. His analysis of the patterns on a tomb in Maragha, Iran, built in 1197, concluded that they map onto Penrose tiles and was published in a 1992 book about fivefold symmetry. Lu and Steinhardt cite his work, he says, but "without proper quoting and … in a way that [the ideas] look like their own."

Physicist Dov Levine of the Israel Institute of Technology in Haifa agrees that Makovicky deserves more credit than he is given in the paper. "His analysis of [the Maragha tomb] patterns anticipates some of the ideas in the Lu and Steinhardt paper," he says. Joshua Socolar, a physicist at Duke University in Durham, North Carolina, agrees that Makovicky deserves credit for discovering "an interesting relation between the Maragha pattern and a Penrose tiling with a few defects." Both Levine and Socolar doubt that the architects truly understood quasi-crystals but say Lu and Steinhardt have generated interesting and testable hypotheses.

Lu and Steinhardt say they were aware of Makovicky's published work on the subject, but "we have found serious problems with both his technical reconstruction and general conclusions." They say that they decided to limit their references to Makovicky "to avoid having to address the serious technical problems with his work." Makovicky disagrees that his work is flawed.

Beyond the question of credit, just how mathematically sophisticated these medieval architects really were remains open. "We haven't done an exhaustive search of Islamic architecture by any means," says Lu. "There could be a perfect quasi-crystal pattern waiting to be found."

–**JOHN BOHANNON**

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